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Multi-criteria optimization of metal pressure processes using experiment planning methods

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Abstract. The main task of designing a technology of metal processing by pressure is accurate prediction and ensuring the desired properties of the finished product. However, they often encounter a situation when it is impossible to simultaneously ensure the optimal value of all process parameters and the resulting properties of the finished product. In this case, it is necessary to make an informed decision regarding the assignment of certain numerical values of these parameters. This article proposes to use the mathematical apparatus of experimental planning methods to implement multi-criteria optimization of metal pressure processes. Applying of correlation analysis, function of desirability and a priori ranking by the example of optimization of parameters of the process of hydromechanical extrusion of hard-to-deform alloys is shown. Results of multi-criteria optimization of the process of hydromechanical extrusion of bar blanks from aluminum alloy AMg5 (analog of Al-5056) with the purpose of minimizing the extrusion force, deformation non-uniformity and damage index of the resulting product are presented.

1. Introduction

At designing of technology of metal processing by pressure it is necessary to deal with a situation when it is impossible to provide simultaneously optimum value of all parameters of process and obtained properties of a finished product. In this case it is necessary to make a reasonable decision concerning assignment of certain numerical values of these parameters, and also possibility of prediction of received properties of a product. One of the methods for solving this type of problems is using methods of experiment planning. Let's apply them for the solution of an existing problem of production of high-precision profiles with high purity of a surface of metals sticking on the tool at which high non-uniformity of deformation on section and length of a product [1]. To eliminate the non-uniformity of deformation it is necessary to reduce the contact friction forces on the working surfaces of the container and matrix [2], or increase the total degree of deformation [3], or increase the plasticity of the deformed material in the pressing process. [4], [5], [6].

At hydromechanical extrusion (HME), besides the pressure of the working fluid, the deformed metal is affected by additional force from the press punch [6]. For high-melting, radioactive or powder materials pressing in a flexible metal shell is used [7], [8].

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To the present time the issues of hydrodynamics of the working flow, which transfers the pressure on the workpiece [9], as well as the influence of tool geometry [10], [11], have been studied quite fully. The results of the experiments are given in the papers [8], [11]–[13].

Experimental and analytical methods [14], [15], as well as simulation modeling with the use of finite element method are used to study the HME process [16].

2. Conducting research

The feature of the considered process is the coverage of the deformable part of the workpiece with a visco-plastic medium that creates high hydrostatic pressure in the cavity of the deforming matrix. The HME process provides the transition of the boundary friction into the semi-liquid friction mode with a corresponding reduction of extrusion pressure.

We solve the extreme task of searching for such a ratio of input parameters of the HME process, at which at the output of the system we get the optimal value of output parameters.

Independent variables are inputted $x_1, x_2, \dots , x_k$.

Outputs $y_1, y_2, \dots , y_m$.

Input and output parameters are related approximately by the following response functions:

$$y_1 = f_1(x_1, x_2, \dots , x_k);$$

$$y_2 = f_2(x_1, x_2, \dots , x_k);$$

$$\dots$$

$$y_m = f_m(x_1, x_2, \dots , x_k).$$

These equations form the factor space of the solved problem, which is reduced to finding the extremum of the response surface.

Let us present a mathematical model to describe the response functions as a Taylor series at any point from the determination area in the factor space. Let us limit to the polynomial of the 2nd degree.

In connection with the specificity of the task, we will construct selective estimates of y as the following regression equation based on the results of the passive experiment:

$$y = b_0 + \Sigma b_i x_i + \Sigma b_{ij} x_i x_j + \Sigma b_{ii} x_i^2 + \dots,$$

where b_0, $b_i = \dfrac{\partial y}{\partial x_i}$, $b_{ij} = \dfrac{\partial^2 y}{\partial x_i \partial x_j}$, $b_{ii} = \dfrac{\partial^2 y}{\partial x_i^2}$, … – selective estimates of regression equation coefficients found from the experiments,

i, j – indexes that meet the following proportion $1 \le i < j \le k$.

Since the multi-criteria optimization task is being solved, we use correlation analysis to establish statistical relationships between parameters.

Let's find the pair correlation coefficients $r_{y_1 y_2}$ between two random variables y_1 and y_2 using next equation:

$$r_{y_1 y_2} = \frac{\sum_{u=1}^{N} \left(y_{1_u} - \overline{y_1}\right)\left(y_{2_u} - \overline{y_2}\right)}{\sqrt{\sum_{u=1}^{N} \left(y_{1_u} - \overline{y_1}\right)^2 \left(y_{2_u} - \overline{y_2}\right)^2}},$$

where N – number of experiments;

u – experiment number;

$$\overline{y_1} = \frac{\sum_{u=1}^{N} y_{1_u}}{N};\ \overline{y_2} = \frac{\sum_{u=1}^{N} y_{2_u}}{N}.$$

After calculation of the pair correlation coefficients $r_{y_1 y_2}$ set their statistical significance on the basis of the critical value of the correlation coefficient r_{cr} by correlation coefficient distribution tables at the selected significance level α and the number of degrees of freedom $f = N - 2$. A linear connection will be considered statistically significant in the case of $|r_{y1y2}| \geq r_{cr}$.

After establishing statistically significant correlation relationships between a pair of optimization parameters y_1 and y_2 it is possible to construct the linear regression equation that allows to predict one parameter y_2 by the value of another y_1:

$$y_2 = b_0 + b_1 y_1,$$

where

$$b_0 = \frac{\sum_{u=1}^{N} y_{2_u} \sum_{u=1}^{N} y_{1_u}^2 - \sum_{u=1}^{N} y_{1_u} \sum_{u=1}^{N} y_{1_u} y_{2_u}}{N \sum_{u=1}^{N} y_{1_u}^2 - \left(\sum_{u=1}^{N} y_{1_u} \right)^2},\ b_1 = \frac{\sum_{u=1}^{N} y_{1_u} y_{2_u} - \sum_{u=1}^{N} y_{1_u} \sum_{u=1}^{N} y_{2_u}}{N \sum_{u=1}^{N} y_{1_u}^2 - \left(\sum_{u=1}^{N} y_{1_u} \right)^2}$$

are coefficients of regression equation.

We construct the revealed linear relationships between the variables in the form of a graph, where the nodes will be the output parameters of the system y, and the edges will be the values of their statistically significant relationship $r_{y_i y_j}$. Solve the leader's task when it is required to determine the influence of the graph node and its power. Influence will be determined by the number of edges that come out of the given node, and the power depends on the influence of other nodes that are connected with the given node. Let's build an adjacency matrix based on the matrix of absolute values of correlation coefficients $\left| r_{y_1 y_2} \right|$.

Find the common element of the adjacency matrix $p^i_j(k)$, defining the number of paths of length k coming from the i-th node to the j-th node.

The iterated k-th order power of the i-th node $p^i_1(k)$ can be found by adding the elements of the adjacent matrix on the lines according to the following formula:

$$p^i(k) = p^i_1(k) + p^i_2(k) + \ldots + p^i_l(k),$$

where l – number of graph's nodes.

Thus, the iterated 1st order power is based on the following formula:

$$\left. \begin{array}{c} p^1(1) = r_{11} + r_{12} + \cdots + r_{1l} \\ p^2(1) = r_{21} + r_{22} + \cdots + r_{2l} \\ \vdots \\ p^l(1) = r_{l1} + r_{l2} + \cdots + r_{ll} \end{array} \right\}.$$

However, the calculation is done only for statistically significant correlation coefficients. Having sorted the obtained values by $p^i(1)$ value, the first approximation of the power estimation of the graph nodes is plotted.

The 2nd order power evaluation matrix is built using the following formula:

$$\left.\begin{array}{c} p^1(2)=r_{11}p^1(1)+r_{12}p^2(1)+\cdots+r_{1l}p^l(1) \\ p^2(2)=r_{21}p^1(1)+r_{22}p^2(1)+\cdots+r_{2l}p^l(1) \\ \vdots \\ p^l(2)=r_{l1}p^1(1)+r_{l2}p^2(1)+\cdots+r_{ll}p^l(1) \end{array}\right\}$$

If after the second approximation of the places distribution the power variables have changed, it is necessary to calculate the third iteration and make analogous further calculations.

After analyzing the power range, the most powerful or influential characteristic is selected as the leader, or an adjustment is made in order to select an output variable that is easier to define methodically.

Then, according to the graph, we build linear regression equations that allow us to determine other parameters by a given value of this output variable.

As far as it is impossible to provide the optimal value of all optimization parameters at the same time, we will understand by desirability d the desired level of any optimization parameter. Numerically d changes from 0 at an unacceptable quality level to 1 at its maximum possible value. The range from 0 to 0.37 corresponds to an unacceptable quality level, from 0.37 to 0.6 to an acceptable and sufficient level, from 0.6 to 0.8 to an acceptable and good quality level and from 0.8 to 1 to a very high quality level. The boundaries of the ranges can be shifted depending on the specific requirements.

For each optimization parameter or y_i response the value of y_i achievable with these input parameters is translated into the desirability function d_i by the following formulas:

one-sided constraint $y \le y_{\text{Max}}$ or $y \ge y_{\min}$:

$$d_i = e^{-e^{-y'_i}},$$

where $y`_i$ – some dimensionless value associated with y_i. More often

$$y'_i = \frac{2y_i - (y_{\max} + y_{\min})}{y_{\max} - y_{\min}}$$

is taken;

two-sided constraint $y_{\min} \le y \le y_{\max}$:

$$d_i = e^{-e^{-(|y'_i|)^{n_i}}},$$

where n_i – a positive number that determines the curvature of the desirability curve.

Approximately n_i can be determined from the ratio:

$$n_i = \frac{\ln\left(\ln \frac{1}{d_i}\right)}{\ln|y'_i|},$$

where d_i – desired level of desirability for this parameter y_i, $0{,}6 < d_i < 0{,}8$.

This mathematical model is programmed in a software complex based on a modified intellectual system [17], [18].

3. Conclusions

We will optimize the HME process of cylindrical workpieces 9 mm in diameter and 22 mm long from aluminum alloy AMg5 (analog of Al-5056). As visco-plastic environment for creation of high hydrostatic pressure we use wax, soap and plastoparaffin [19]. Rheological properties of plastoparaffin are given in [20], [21].

Optimization was carried out based on the results of the experiment, as shown in table 1.

As a result of the simulation HME process provides a compromise solution to the problem of minimizing the extrusion force, non-uniformity of the deformed state and damage index to the finished product.

Application of the program complex made it possible to determine the dependence of the overall quality criterion of the HME process on the main technological parameters of the HME: drawing ratio λ, conicity angle of matrix α and visco-plastic coating parameters of the extruded workpiece.

As a result of multi-criteria optimization of the HME process of an aluminum alloy, the following values of technological parameters are recommended: drawing ratio $\lambda = 4.8$; matrix conicity angle $\alpha = 48^o$, as a environment creating hydrostatic pressure in the matrix cavity, a soap powder ensuring friction coefficient $f = 0.05$ can be used.

Table 1. Experimental data for the HME process of AMg5 alloy.

	Independent variables			Responses		
	Drawing ratio λ	Matrix angle, α, degree	Friction coefficient, f	Extrusion force x10^4, N	Damage index	Deformation uniformity
№	x_1	x_2	x_3	y_1	y_2	y_3
1	2	50	0.3	2.57	0.27	2.95
2	4	55	0.3	5.04	0.18	2.19
3	6	60	0.3	6.13	0.17	2.12
4	8	60	0.4	7.60	0.18	1.96
5	2	60	0.1	2.62	0.31	3.89
6	4	60	0.2	4.92	0.31	2.44
7	8	50	0.2	6.34	0.17	1.66
8	6	50	0.1	5.29	0.77	1.77
9	8	55	0.1	6.10	0.18	1.79

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